

Role of Three-Qubit Mixed-States Entanglement in Teleportation Scheme

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Abstract

The bipartite quantum teleportation with a three-qubit mixture of GHZ and W states as a quantum channel is discussed. When the quantum channel is a mixed state consisting of the GHZ state plus small perturbed W state, the entanglement of the channel becomes zero when the average fidelity \bar{F}_{GHZ} is less than 0.775. This means that the mixed state cannot play an any role as a quantum channel at $\bar{F}_{GHZ} \leq 0.775$. For the case of the mixed state consisting of the W state plus small perturbed GHZ state it turn out that this mixed state cannot play a role as a quantum channel at $\bar{F}_W \leq 0.833$.

I. INTRODUCTION

Recently, there has been a flurry of activity in the entanglement of the quantum states[1]. It seems to be a genuine physical resource which makes quantum computer outperforms classical one[2]. It plays an important role in other branches of physics. For example, entanglement may give a promising approach to understand the information loss problem of the black hole physics. The entanglement between matters and gravity[3] may shed light on the various difficulties arising from the black hole physics. Therefore, it is highly important to understand the general properties of the quantum entanglement in the context of quantum information theories[4].

The quantum entanglement for the two-qubit states is well understood regardless of pure or mixed states. For example, the entanglement of formation \mathcal{E} [5] and the Groverian measure G [6], two of the basic entanglement measures, for any two-qubit pure states can be straightly computed from the concurrence \mathcal{C} using a formula

$$\mathcal{E}(\psi) = h\left(\frac{1 + \sqrt{1 - \mathcal{C}^2(\psi)}}{2}\right) \quad G(\psi) = \frac{1}{\sqrt{2}} \left[1 - \sqrt{1 - \mathcal{C}^2(\psi)}\right]^{1/2} \quad (1.1)$$

where $h(x) \equiv -x \log_2 x - (1 - x) \log_2 (1 - x)$. For two-qubit state $|\psi\rangle = \sum_{i,j=0}^1 a_{ij}|ij\rangle$ the concurrence $\mathcal{C}(\psi)$ becomes

$$\mathcal{C}(\psi) = 2|a_{00}a_{11} - a_{01}a_{10}|. \quad (1.2)$$

Thus, it is maximal for Bell states and vanishes for factorized states. Combining (1.1) and (1.2), one can compute the entanglement of formation and the Groverian measure for all two-qubit pure states.

The entanglement for the mixed states is in general defined by making use of the convex roof construction[7, 8]¹. For example, the concurrence for the two-qubit mixed state ρ is defined as

$$\mathcal{C}(\rho) = \min \sum_i p_i \mathcal{C}(\rho_i) \quad (1.3)$$

where minimum is taken over all possible ensembles of pure states. The ensemble which gives the minimum value in Eq.(1.3) is called the optimal decomposition of the mixed state ρ . Few years ago W. K. Wootters[10, 11] has shown how to construct the optimal ensembles for the

¹ For Groverian measure of mixed states there is an entanglement monotone, which does not follow the convex roof construction. See Ref.[9]

arbitrary two-qubit mixed states by considering the time reversal operation of the spin-1/2 particles. Making use of these optimal decompositions, one can analytically compute the concurrence for all two-qubit states by a simple formula

$$\mathcal{C}(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4) \quad (1.4)$$

where λ_i 's are the eigenvalues, in decreasing order, of the Hermitian matrix

$$\sqrt{\sqrt{\rho}(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)\sqrt{\rho}}.$$

Note that $\mathcal{E}(\psi)$ and $G(\psi)$ in Eq.(1.1) are monotonic functions with respect to $\mathcal{C}(\psi)$. This fact indicates that the optimal decompositions for the entanglement of formation and the Groverian measure are same with those for the concurrence. Thus one can compute \mathcal{E} , G and \mathcal{C} for all two-qubit states regardless of pure or mixed.

Recently, \mathcal{E} , G and \mathcal{C} for the various mixed states arising in the teleportation process through noisy channels have been explicitly computed[12]. Due to the noises the sender, Alice, cannot send the single-qubit state $|\psi_{in}\rangle$ to the receiver, Bob, perfectly. If Bob receives ρ_{out} , one can compute $F \equiv \langle \psi_{in} | \rho_{out} | \psi_{in} \rangle$, which measures how well the teleportation job is performed. It is shown in Ref.[12] that the mixed states entanglements \mathcal{E} , G and \mathcal{C} all vanish when the average of F , say \bar{F} , is less than 2/3, which corresponds to the best possible score when Alice and Bob communicate with each other through classical channel[13]. This fact implies that the mixed state entanglement is a genuine physical resource for the teleportation process through noisy channels.

In this paper we would like to explore same issue with Ref.[12] in the teleportation when the quantum channel is three-qubit mixed states. It is well-known that not only two-qubit Bell state but also three-qubit GHZ[14] and W[15] states

$$|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \quad |\psi_W\rangle = \frac{1}{2}(|100\rangle + |010\rangle + \sqrt{2}|001\rangle) \quad (1.5)$$

allow the perfect teleportation if Alice and Bob share the qubits appropriately at the initial stage.. The imperfect teleportation due to various noises was discussed in Ref.[16] when GHZ and W states are prepared for two-party quantum teleportation through a noisy channel. The various three-qubit mixed states and the average fidelity \bar{F} were explicitly derived in the reference. It has been shown that the issue of robustness between GHZ and W, i.e. which state does lose less quantum information, in the noisy channels is completely dependent on the type of noisy channel.

Then, it is natural to ask the following questions. Does the entanglement for the three-qubit mixed state play an important role in the teleportation process as that for the two-qubit mixed state does as shown in Ref.[12]? What is the relation between \bar{F} and entanglement? Do all entanglement measures vanish when $\bar{F} \leq 2/3$? The purpose of this paper is to try to address these questions.

However, unfortunately, our knowledge on the entanglement of the three-qubit mixed state is extremely limited. Especially, the entanglement of formation and the concurrence are in fact defined in the bipartite system. Therefore, we cannot use them in the three-qubit state². Of course, the Groverian measure can be defined in the arbitrary qubit states. Although, recently, the analytical computation of the Groverian measure for several three-qubit pure states was done[19, 20, 21, 22], still we do not know how to construct the optimal decomposition for the three-qubit mixed states to compute the Groverian measure. Since, however, we are considering the two-party quantum teleportation with three-qubit mixed state and the sender, Alice, has usually first two qubits, it is sufficient to consider the two-party concurrence $\mathcal{C}_{(AB)C}$ if ρ^{ABC} is a given mixed state. The concurrence $\mathcal{C}_{(AB)C}$ can be computed for three-qubit pure states by making use of the three-tangle[23]. Furthermore, although we can not compute it for arbitrary three-qubit mixed state, we know how to construct the optimal decomposition at least for the mixtures of GHZ and W states[24, 25]. Thus, we can use these mixture states to address the purpose of the present paper.

This paper is organized as follows. In section II we would like to briefly review the three-tangle and its optimal decomposition for the mixtures of GHZ and W states. In this section we will compute $\mathcal{C}_{(AB)C}$ explicitly for the mixed state we will use in the teleportation process. In section III we will compute the average fidelity \bar{F} when the mixture consisting of the unperturbed GHZ state and small perturbed W state is used as a quantum channel. It is shown that the entanglement of the quantum channel becomes zero at $\bar{F} \leq 0.775$. This fact indicates that the bipartite teleportation with GHZ state is more unstable than that with usual EPR state under the perturbed interaction. In section IV we will compute the average fidelity \bar{F} when the mixture consisting of the unperturbed W state and small perturbed GHZ state is used as a quantum channel. unperturbed W and small perturbed GHZ states is used as a quantum channel. It is shown that the entanglement of the quantum channel

² Recently, there was a try to extend the definition of the concurrence to the multi-qubit system.[17, 18]

becomes zero at $\bar{F} \leq 0.833$. In section V a brief discussion on the previous calculational results is given.

II. THREE-TANGLE AND COMPUTATION OF $\mathcal{C}_{(AB)C}$

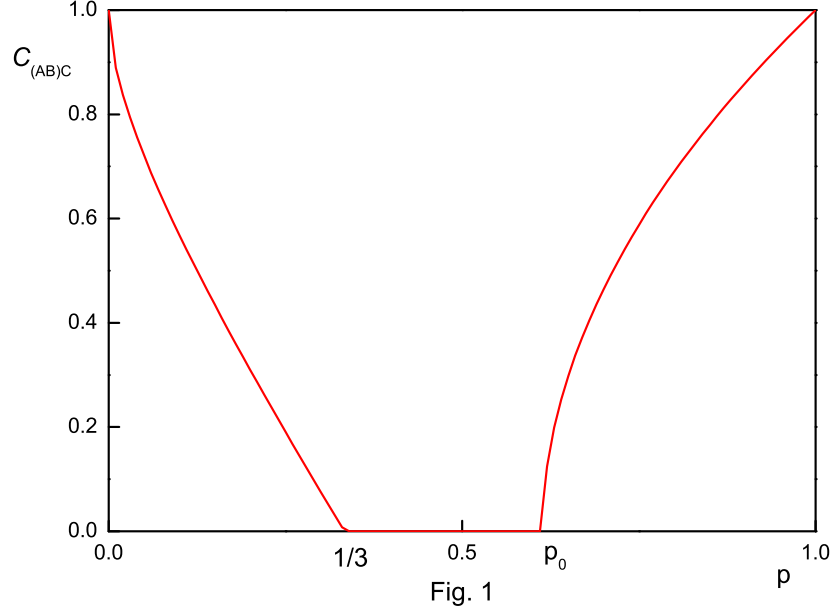


FIG. 1: Plot of p -dependence of $\mathcal{C}_{(AB)C}$. The non-zero of $\mathcal{C}_{(AB)C}$ at $p > p_0$ is mainly due to the three-tangle of the mixed state ρ^{QC} while the remaining non-vanishing value at $p < 1/3$ is due to the concurrences of the reduced states. The fact that $\mathcal{C}_{(AB)C} = 1$ at $p = 0$ and $p = 1$ implies that pure GHZ and pure W states are maximally entangled.

For three-qubit pure state ρ^{ABC} the concurrences \mathcal{C}_{AC} and \mathcal{C}_{BC} for the reduced states ρ^{AC} and ρ^{BC} satisfy the following inequality[23]

$$\mathcal{C}_{AC}^2 + \mathcal{C}_{BC}^2 \leq \mathcal{C}_{(AB)C}^2 \quad (2.1)$$

where $\mathcal{C}_{(AB)C}$ is a concurrence between a pair AB and the qubit C. The difference between rhs and lhs in Eq.(2.1) is defined as a three-tangle or residual entanglement:

$$\tau_{ABC} \equiv \mathcal{C}_{(AB)C}^2 - (\mathcal{C}_{AC}^2 + \mathcal{C}_{BC}^2). \quad (2.2)$$

For the state $|\psi\rangle = \sum_{i,j,k=0}^1 a_{ijk}|ijk\rangle$, τ_{ABC} becomes[23]

$$\tau_{ABC} = 4|d_1 - 2d_2 + 4d_3| \quad (2.3)$$

where

$$\begin{aligned} d_1 &= a_{000}^2 a_{111}^2 + a_{001}^2 a_{110}^2 + a_{010}^2 a_{101}^2 + a_{100}^2 a_{011}^2 \\ d_2 &= a_{000} a_{111} a_{011} a_{100} + a_{000} a_{111} a_{101} a_{010} + a_{000} a_{111} a_{110} a_{001} \\ &\quad + a_{011} a_{100} a_{101} a_{010} + a_{011} a_{100} a_{110} a_{001} + a_{101} a_{010} a_{110} a_{001} \\ d_3 &= a_{000} a_{110} a_{101} a_{011} + a_{111} a_{001} a_{010} a_{100}. \end{aligned} \quad (2.4)$$

Thus the generalized GHZ and W states defined

$$|GHZ\rangle = a|100\rangle + b|111\rangle \quad |W\rangle = c|001\rangle + d|010\rangle + f|100\rangle \quad (2.5)$$

have

$$\tau_3^{GHZ} = 4|a^2 b^2| \quad \tau_3^W = 0. \quad (2.6)$$

For the mixed three-qubit state ρ the three-tangle is defined by

$$\tau_3(\rho) = \min_j \sum_j p_j \tau_3(\rho_j) \quad (2.7)$$

where minimum is taken over all possible ensembles of pure states. Thus we should construct the optimal decomposition to compute Eq.(2.7). The construction of the optimal decompositions for the arbitrary three-qubit mixed states is an highly nontrivial and formidable job and is yet unsolved. However, for the mixture of GHZ and W states given by

$$\rho(p) = p|GHZ\rangle\langle GHZ| + (1-p)|W\rangle\langle W| \quad (2.8)$$

the optimal decompositions has been constructed in Ref.[24, 25]. The authors in these references used a fact that the state

$$|p, \varphi\rangle = \sqrt{p}|GHZ\rangle - \sqrt{1-p}e^{i\varphi}|W\rangle \quad (2.9)$$

has nontrivial zero of the three-tangle at

$$\varphi = n\frac{2\pi}{3} \quad (n \in \mathbb{N}) \quad p = p_0 = \frac{s^{2/3}}{1 + s^{2/3}} \quad (2.10)$$

where $s = 4cdf/a^2b$.

When $p \leq p_0$ one can derive an optimal ensemble in the form

$$\rho(p) = \frac{p}{3p_0} \left[|p_0, 0\rangle\langle p_0, 0| + |p_0, \frac{2\pi}{3}\rangle\langle p_0, \frac{2\pi}{3}| + |p_0, \frac{4\pi}{3}\rangle\langle p_0, \frac{4\pi}{3}| \right] + \left(1 - \frac{p}{p_0}\right) |W\rangle\langle W| \quad (2.11)$$

which gives the vanishing three-tangle. The remaining optimal decompositions at $p > p_0$ have been derived in Ref.[24, 25] explicitly, and we do not want to repeat the derivation here. What we want to do here is to summarize the three-tangle as follows:

$$\tau_3(\rho(p)) = \begin{cases} 0 & \text{for } 0 \leq p \leq p_0 \\ \tau_3(p) & \text{for } p_0 \leq p \leq p_1 \\ \tau_3^{conv}(p) & \text{for } p_1 \leq p \leq 1 \end{cases} \quad (2.12)$$

where

$$\begin{aligned} \tau_3(p) &= \tau_3^{GHZ} |p^2 - \sqrt{p(1-p)^3} s| \\ \tau_3^{conv}(p) &= \tau_3^{GHZ} \left[\frac{p-p_1}{1-p_1} + \frac{1-p}{1-p_1} \left(p_1^2 - \sqrt{p_1(1-p_1)^2} s \right) \right] \end{aligned} \quad (2.13)$$

and

$$p_0 = \frac{s^{2/3}}{1+s^{2/3}} \quad p_1 = \max \left(p_0, \frac{1}{2} + \frac{1}{2\sqrt{1+s^2}} \right). \quad (2.14)$$

As we mentioned in the previous section we will consider in this paper the quantum teleportation with a quantum channel

$$\rho^{QC} = p |\psi_{GHZ}\rangle\langle\psi_{GHZ}| + (1-p) |\psi_W\rangle\langle\psi_W| \quad (2.15)$$

where $|\psi_{GHZ}\rangle$ and $|\psi_W\rangle$ are given in Eq.(1.5). Comparing Eq.(2.8) with Eq.(2.15), we have $a = b = c = 1/\sqrt{2}$ and $d = f = 1/2$, which give

$$s = 2, \quad \tau_3^{GHZ} = 1, \quad p_0 = \frac{2^{2/3}}{1+2^{2/3}} \sim 0.614, \quad p_1 = \frac{1+\sqrt{5}}{2\sqrt{5}} \sim 0.724. \quad (2.16)$$

Thus the three-tangle for ρ^{QC} becomes

$$\tau_3(\rho^{QC}) = \begin{cases} 0 & \text{for } 0 \leq p \leq p_0 \\ |p^2 - 2\sqrt{p(1-p)^3}| & \text{for } p_0 \leq p \leq p_1 \\ \frac{1}{1-p_1} [(1-t_1)p - (p_1-t_1)] & \text{for } p_1 \leq p \leq 1 \end{cases} \quad (2.17)$$

where $t_1 \equiv p_1^2 - 2\sqrt{p_1(1-p_1)^3} \sim 0.276$.

Since it is simple to derive the reduced states from ρ^{QC} , one can easily compute the concurrences \mathcal{C}_{AB} , \mathcal{C}_{AC} and \mathcal{C}_{BC} following Wootters procedure[10, 11], whose explicit expressions are

$$\begin{aligned}\mathcal{C}_{AB} &= \begin{cases} \frac{1-p-2\sqrt{p}}{2} & \text{for } 0 \leq p \leq 3-2\sqrt{2} \\ 0 & \text{for } 3-2\sqrt{2} \leq p \leq 1 \end{cases} \\ \mathcal{C}_{AC} = \mathcal{C}_{BC} &= \begin{cases} \frac{1}{\sqrt{2}} \left[(1-p) - \sqrt{p(1+p)} \right] & \text{for } 0 \leq p \leq \frac{1}{3} \\ 0 & \text{for } \frac{1}{3} \leq p \leq 1 \end{cases}\end{aligned}\quad (2.18)$$

Thus one can compute $\mathcal{C}_{(AB)C}$ as

$$\mathcal{C}_{(AB)C} = \sqrt{\mathcal{C}_{AC}^2 + \mathcal{C}_{BC}^2 + \tau_3(\rho^{QC})}. \quad (2.19)$$

The p -dependence of $\mathcal{C}_{(AB)C}$ is plotted in Fig. 1. This figure shows that $\mathcal{C}_{(AB)C} = 1$ at $p = 0$ and $p = 1$, which indicates that pure GHZ and pure W states are maximally entangled. This fact also indicates that the two-party teleportation with $p = 0$ and $p = 1$ states will be perfect. This will be confirmed in next section by showing that the average fidelity \bar{F} becomes unit at these points. The non-vanishing $\mathcal{C}_{(AB)C}$ at $p < 1/3$ is due to the concurrences of the reduced states while the remaining non-vanishing value at $p > p_0$ is due to the three-tangle. This fact indicates that the entanglement of pure GHZ states mainly comes from the three-tangle while that of pure W state comes from the entanglement of its reduced states.

III. QUANTUM TELEPORTATION WITH LARGE p STATE

In this section we consider the quantum teleportation with a mixed state ρ^{QC} given in Eq.(2.15) when p is large. The state ρ^{QC} with large p can be regarded as a mixed state which consists of the GHZ state plus small perturbed W state. Therefore, we use a teleportation scheme with GHZ state, whose quantum circuit is given in Fig. 2.

Now, we assume that the sender, called Alice, who has first two qubits in ρ^{QC} , wants to send a single qubit

$$|\psi_{in}\rangle = \cos\left(\frac{\theta}{2}\right) e^{i\phi/2}|0\rangle + \sin\left(\frac{\theta}{2}\right) e^{-i\phi/2}|1\rangle \quad (3.1)$$

to the receiver, called Bob, who has last qubit in ρ^{QC} . Then Fig. 2 implies that the state ρ_{out} , which Bob has finally, becomes

$$\rho_{out} = \text{Tr}_{1,2,3} \left[U_{GHZ} (\rho_{in} \otimes \rho^{QC}) U_{GHZ}^\dagger \right] \quad (3.2)$$

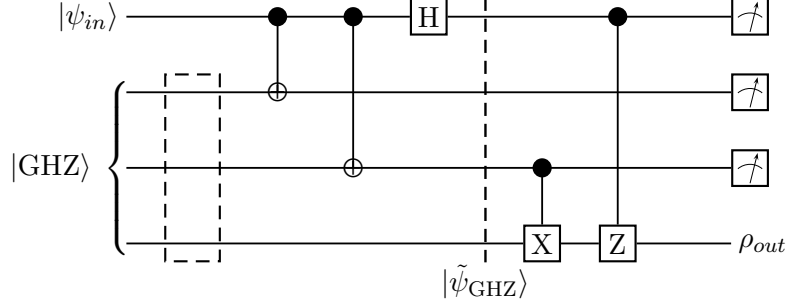


FIG. 2: A quantum circuit for quantum teleportation through noisy channels with GHZ state. The top three lines belong to Alice while the bottom line belongs to Bob. The dotted box represents small perturbation, which makes the quantum channel to be mixed state.

where $\text{Tr}_{1,2,3}$ is partial trace over Alice's qubits and $\rho_{in} = |\psi_{in}\rangle\langle\psi_{in}|$. The unitary operator U_{GHZ} can be read directly from Fig. 2 and its explicit expression is

$$U_{GHZ} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (3.3)$$

Inserting Eq.(2.15), Eq.(3.1), and Eq.(3.3) into Eq.(3.2), one can compute ρ_{out} straightforwardly.

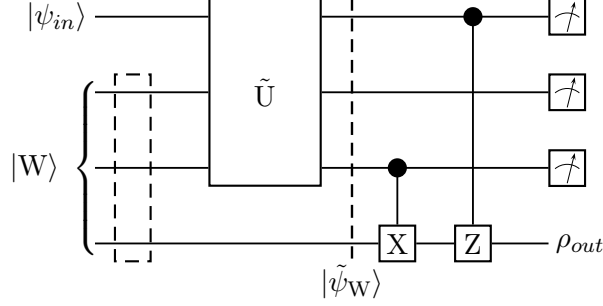


FIG. 3: A quantum circuit for quantum teleportation through noisy channels with W state. The top three lines belong to Alice while the bottom line belongs to Bob. The dotted box represents small perturbation, which makes the quantum channel to be mixed state. The unitary operator \tilde{U} makes $|\tilde{\psi}_W\rangle$ coincide with $|\tilde{\psi}_{GHZ}\rangle$ expressed in Fig. 2

In order to quantify how much information is preserved or lost during teleportation we consider a quantity

$$F(\theta, \phi) = \langle \psi_{in} | \rho_{out} | \psi_{in} \rangle, \quad (3.4)$$

which is the square of the usual fidelity defined $F(\rho, \sigma) = \text{Tr} \sqrt{\rho^{1/2} \sigma \rho^{1/2}}$. Thus, $F = 1$ implies the perfect teleportation.

For our case $F_{GHZ}(\theta, \phi)$ becomes

$$F_{GHZ}(\theta, \phi) = \frac{1}{8} [(3 + 5p) - (1 - p) \cos(2\theta)]. \quad (3.5)$$

When $p = 1$, F_{GHZ} becomes one. This means that the pure GHZ state allows the perfect teleportation.

Now we define the average fidelity in a form

$$\bar{F} \equiv \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta F(\theta, \phi). \quad (3.6)$$

For our case \bar{F}_{GHZ} becomes

$$\bar{F}_{GHZ} = \frac{5 + 7p}{12}. \quad (3.7)$$

When $p = 1$, \bar{F}_{GHZ} becomes one again.

IV. QUANTUM TELEPORTATION WITH SMALL p STATE

Since the state ρ^{QC} in Eq.(2.15) with small p can be regarded as a mixed state which consists of the W state plus small perturbed GHZ state, we use a teleportation scheme with

a pure W states. The quantum circuit for this scheme is given in Fig.3. In this figure the unitary operator \tilde{U} is introduced to make $|\tilde{\psi}_W\rangle$ to be same with $|\tilde{\psi}_{GHZ}\rangle$ in Fig. 2. The explicit expression of \tilde{U} is given in Eq.(3.1) of Ref.[16].

The final state ρ_{out} which Bob has finally becomes

$$\rho_{out} = \text{Tr}_{1,2,3} \left[U_W (\rho_{in} \otimes \rho^{QC}) U_W^\dagger \right] \quad (4.1)$$

where $\text{Tr}_{1,2,3}$ is partial trace over Alice's qubits and $\rho_{in} = |\psi_{in}\rangle\langle\psi_{in}|$. The explicit expression of U_W can be read directly from Fig. 3 in the form:

$$U_W = \frac{1}{2} \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & -\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{2} & 0 & -\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{2} & 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2} & 0 & -\sqrt{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sqrt{2} & 0 & \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 \\ -\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad (4.2)$$

Then $F_W(\theta, \phi)$ and \bar{F}_W can be straightforwardly computed. It is shown that $F_W(\theta, \phi)$ is independent of the angle parameters. Thus $F_W(\theta, \phi)$ is same with the average fidelity in a form:

$$\bar{F}_W = F_W(\theta, \phi) = 1 - \frac{p}{2}. \quad (4.3)$$

When $p = 0$, \bar{F}_W becomes one, which indicates that the pure W state allows the perfect teleportation.

V. DISCUSSION

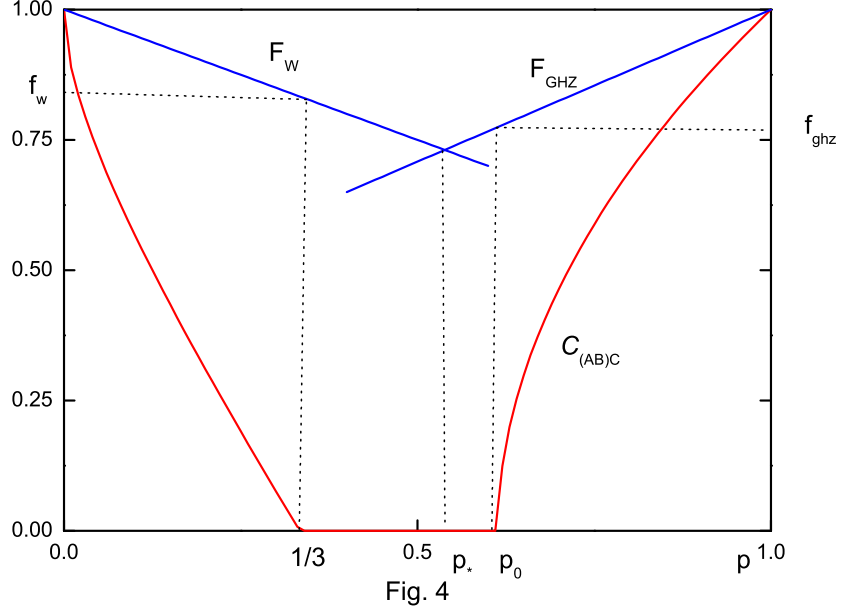


FIG. 4: Plot of p -dependence of \bar{F}_{GHZ} and \bar{F}_W . For comparison we plot the p -dependence of $\bar{C}_{(AB)C}$ together. For the case of GHZ with small perturbed W state the entanglement becomes zero at $\bar{F}_{GHZ} < f_{ghz} \sim 0.775$. For the case of W with small perturbed GHZ state $\bar{C}_{(AB)C}$ becomes zero at $\bar{F}_W < f_w \sim 0.833$. In this aspect we can say that GHZ state is more robust in the teleportation process than W state under the perturbed interaction.

Fig. 4 is a plot of p -dependence of \bar{F}_{GHZ} and \bar{F}_W together with $\bar{C}_{(AB)C}$. As expected, \bar{F}_{GHZ} and \bar{F}_W become unit when $p = 1$ and $p = 0$, respectively. At $p = p_0$ \bar{F}_{GHZ} becomes f_{ghz} , where

$$f_{ghz} = \frac{1}{12}(5 + p_0) \sim 0.774549. \quad (5.1)$$

When $\bar{F}_{GHZ} \leq f_{ghz}$, $\bar{C}_{(AB)C}$ of ρ^{QC} with large p becomes zero. This means that the mixed state consisting of the GHZ plus small perturbed W state cannot play a role as a quantum channel in the teleportation process when $\bar{F}_{GHZ} \leq f_{ghz}$. In this sense f_{ghz} is something like the critical value in the average fidelity \bar{F}_{GHZ} . It is interesting to note $f_{ghz} > 2/3$, where $\bar{F} = 2/3$ plays same role in the bipartite teleportation through noisy channel when EPR state

is used as a quantum channel[12]. This fact indicates that the bipartite teleportation with GHZ state is much more unstable than teleportation with EPR state under the perturbed interaction.

The critical value of \bar{F}_W is f_w , where

$$f_w = \frac{5}{6} \sim 0.833333. \quad (5.2)$$

Since $f_w > f_{ghz}$, the bipartite teleportation with GHZ state is more stable than that with W state. In this sense we can say that GHZ state is more robust than W state under the perturbation.

There is another point we would like to consider. Fig. 4 shows that $\bar{F}_W \geq \bar{F}_{GHZ}$ in the region $0 \leq p \leq p_* = 7/13 \sim 0.538$. Since $p_* > 1/2$, this means that ρ^{QC} can be regarded as a W state with perturbed GHZ state in the more wide range of p . In this aspect we can say that W state is more robust than GHZ state in the perturbed interaction.

Although we have considered the bipartite teleportation with a three-qubit mixed state ρ^{QC} , this is not a teleportation through noisy channels because ρ^{QC} is not derived from the master equation[26]. Many three-qubit mixed states are explicitly derived in Ref.[16] in the teleportation through noisy channels. For example, the mixed state derived from W state with $(L_{2,x}, L_{3,x}, L_{4,x})$ noisy channel is

$$\varepsilon_x(\rho_W) = \frac{1}{16} \begin{pmatrix} 2\alpha_2 & 0 & 0 & \sqrt{2}\alpha_2 & 0 & \sqrt{2}\alpha_2 & \alpha_2 & 0 \\ 0 & 2\alpha_1 & \sqrt{2}\alpha_1 & 0 & \sqrt{2}\alpha_1 & 0 & 0 & \alpha_3 \\ 0 & \sqrt{2}\alpha_1 & 2\beta_+ & 0 & \alpha_1 & 0 & 0 & \sqrt{2}\alpha_3 \\ \sqrt{2}\alpha_2 & 0 & 0 & 2\beta_- & 0 & \alpha_4 & \sqrt{2}\alpha_4 & 0 \\ 0 & \sqrt{2}\alpha_1 & \alpha_1 & 0 & 2\beta_+ & 0 & 0 & \sqrt{2}\alpha_3 \\ \sqrt{2}\alpha_2 & 0 & 0 & \alpha_4 & 0 & 2\beta_- & \sqrt{2}\alpha_4 & 0 \\ \alpha_2 & 0 & 0 & \sqrt{2}\alpha_4 & 0 & \sqrt{2}\alpha_4 & 2\alpha_4 & 0 \\ 0 & \alpha_3 & \sqrt{2}\alpha_3 & 0 & \sqrt{2}\alpha_3 & 0 & 0 & 2\alpha_3 \end{pmatrix} \quad (5.3)$$

where

$$\begin{aligned}
\alpha_1 &= 1 + e^{-2\kappa t} + e^{-4\kappa t} + e^{-6\kappa t} \\
\alpha_2 &= 1 + e^{-2\kappa t} - e^{-4\kappa t} - e^{-6\kappa t} \\
\alpha_3 &= 1 - e^{-2\kappa t} - e^{-4\kappa t} + e^{-6\kappa t} \\
\alpha_4 &= 1 - e^{-2\kappa t} + e^{-4\kappa t} - e^{-6\kappa t} \\
\beta_{\pm} &= 1 \pm e^{-6\kappa t}.
\end{aligned} \tag{5.4}$$

Unfortunately, still there is no method, as far as we know, for the construction of the optimal decomposition of $\varepsilon_x(\rho_W)$. Thus it is highly nontrivial and formidable task to compute the three-tangle for this mixed state. We would like to develop a computational technique, which enables us to compute the three-tangle for arbitrary three-qubit mixed states or, at least, the various mixed states derived from master equation in Ref.[16]. If then, we may understand more deeply the role of entanglement in the various quantum algorithms.

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